Univerzitet u Beogradu
Publikacije Elektrotehničkog Fakulteta
Serija Tehnička fizika (1997), 24–32

Fokker-Planck Studies of Magnetically Confined Plasmas Part II Quasilinear Electron Cyclotron Heating of Tokamak Plasma

I. Sajić and S. Pešić

"Vinča" Institute for Nuclear Sciences, Laboratory for Nuclear and Plasma Physics, P.O.B. 522, 11001 Belgrade, Yugoslavia

Some aspects of the physics of plasma heating and current drive by electron cyclotron (EC) waves in fusion plasmas are pointed out by analyzing numerically an illustrative plasma heating experiment. The numerical code we have developed to solve the bounce-averaged quasilinear Fokker-Planck equation is an interpretative and predictive code. It incorporates many of the geometrical and physical features encountered in experiments. We found that the EC quasilinear diffusion coefficient is a complex function of the velocity space variables. At high power levels ($P_{inj} \sim 2MW$), large distortions in the electron distribution function are generated. It is shown that the RF driven current density is strongly dependent on the injected power. A saturation of the current density, which is due to the effect of heating out of resonance, occurs at high powers. A deterioration of the current drive efficiency, caused by the interplay of electron trapping and quasilinear effects, is observed.

I Introduction

The interaction of electromagnetic fields or beams of particles with plasma is usually studied by following the evolution of the plasma particle distribution function (DF). Examples include plasma heating by electromagnetic waves or energetic neutral beams, runaway electrons in tokamaks, mirror light- or heavy-ion plasmas, laser produced plasmas, thermalization of α-particles in deuterium-tritium plasma, etc. Generally, under the action of external fields or particle beams the particle DF evolves in the configuration and velocity space. The interaction between particles and waves is localized by the resonance condition involving the particle motion, the fluctuating high-frequency field and the steady magnetic field B,

$$\omega - k_{||}v_{||} = n\omega_c \tag{1}$$

where ω is the wave frequency, $n = 0, \pm1, \pm2, ..., \omega_c = eB/m(v)$ is the electron cyclotron frequency,

$m(v)$ is the relativistic electron mass, $k_{||}$ and $v_{||}$ are the components of the wave vector and the electron velocity, respectively, which are parallel to B. For each type of wave, this condition determines a resonance region $v_1 < v_{||} < v_2$ in velocity space, where an intense interaction between particles and waves arises. Namely, in this velocity range, a group of electrons remains in phase with the wave over many cycles and has time to exchange energy with the wave, resulting in slow changes in the DF, which may be attributed to a diffusion process called quasilinear diffusion. Because of the resonance nature of the interaction, this is a kinetic process, and gives rise to phenomena such as plateau formation in the DF, and general distortions of the DF such as tail formation. The process ceases when plateau in the DF is completely formed. The indicated plateau is only suggestive however, since it was drawn to conserve energy, but fails to conserve momentum. To conserve both, the entire distribution must be shifted by various amounts, and this requires nonresonant diffusion processes. The nonresonant diffusion is much weaker than the resonant one, but it acts on many more particles, so the net effects may be of the same order of magnitude. It should be noted that although the distribution eventually is modified so that it is everywhere stable or marginally stable, it is not an equilibrium distribution and there remain finite amplitude waves in the marginally stable regions of the wave spectrum as a remnant of the originally unstable distribution. It remains for collisional effects to finally relax the DF towards equilibrium.

II Bounce-averaged electron cyclotron quasilinear diffusion

Generally, the flux S_w (see expression (5) in [1]) induced by quasilinear wave-particle interaction can be expressed as,

$$\vec{S}_w = -\hat{D} \cdot \nabla f \qquad (2)$$

The quasilinear diffusion tensor $\hat{D}$ describing particle scattering by waves depends on the type, spectral distribution, and energy density of the oscillations. It is given by,

$$\hat{D} = \sum_n \frac{\pi e^2}{2m^2} \delta(\omega - k_{||}v_{||} - n\omega_c) \vec{a}_n^* \otimes \vec{a}_n \qquad (3)$$

and

$$\vec{a}_n = \theta_n \frac{k_{||}}{\omega} \left[\left(\frac{\omega}{k_{||}} - v_{||} \right) \vec{e}_\perp + v_\perp \vec{e}_{||} \right]$$

$$\theta_n = [E_+ J_{n-1} + E_- J_{n+1}]/\sqrt{2} + v_{||} E_{||} J_n / v_\perp$$

where J_n is the Bessel function of the first kind whose argument is $k_\perp v_\perp / \omega_c$, E_+ and E_- are the left- and right-handed components of the wave field $\vec{E}$ in the right-handed Cartesian coordinate system with the z-axis parallel to $\vec{B}$, and $\vec{k}$ lying in the (x, z) plane,

$$E_+ = (E_x + iE_y)/\sqrt{2}, \quad E_- = (E_x - iE_y)/\sqrt{2}, \quad E_{||} = E_z$$

The delta function in (3) specifies the resonance condition (1). Only particles for which the Doppler-shifted wave frequency $\omega - k_{||}v_{||}$ is zero ($n = 0$ is the Landau resonance) or a multiple of the cyclotron frequency ($n \neq 0$ is a cyclotron harmonic resonance) interact with the wave. The vector $\vec{a}_n$ is perpendicular to the velocity of the particle in the wave frame $\vec{v} - \omega \vec{e}_{||}/k_{||}$. This means that the wave-induced flux is along diffusion paths which lie in constant-energy surfaces in the wave frame. Similarly, the flux S_w is proportional to the gradient in f in this direction. Therefore, when the particles interact with the wave via the cyclotron harmonic resonance, the diffusion tensor consists of only a single component

$$\hat{D} = D_{\perp\perp} \vec{e}_\perp \otimes \vec{e}_\perp \qquad (4)$$

It is instructive to consider the properties of $D_{\perp\perp}$ in the single particle model. In this model, the diffusion coefficient $D_{\perp\perp}$ is found calculating the increment in perpendicular velocity, $\Delta v_\perp$ as a particle passes through the wave beam. The diffusion coefficient is then obtained from the expression,

$$D_{\perp\perp} = \left[(\Delta v_\perp)^2\right]/(2\Delta t) \tag{5}$$

where the square brackets denote the average over the initial gyrophase, and Δt is the mean time between successive transits through the wave beam. Equation (5) is valid if $\Delta v_\perp$ is less than the gradient scale length of the DF in perpendicular velocity.

The appearance of the delta function in (3) is a consequence of the assumed magnetic field uniformity. In this case, $v_{||}$ is a constant of the unperturbed motion and so, the particle remains in resonance for a long time. The magnetic confinement of plasma however, is based on a nearly recurrent particle motion in a nonuniform magnetic field. In fusion plasma devices, the magnetic field and $v_{||}$ vary along a particle orbit so that the particle does not remain in resonance. As discussed in [1] this effect is taken into account by averaging the relevant quantities, in this case the expression (3), over a magnetic flux surface.

Since the microwave beam has a finite width both in poloidal and toroidal directions the averaging procedure should contain two functions: , $_1(\chi)$ which represents the shape of the wave beam in the poloidal cross section and is expressed as a function of the angle χ, and , $_2(\chi)$ which is the $N_{||}$ spectrum of the wave field, related to the beam shape in the toroidal direction via Fourier transform. Before going further, let us specify the spatial dependence of the wave field $\vec{E}(r)$. The injected beam is taken to have a Gaussian profile, which is symmetric about the beam axis, and each ray is assumed to suffer the same absorption in propagating to a given value of the tokamak major radius (see Fig.6 in [1]).

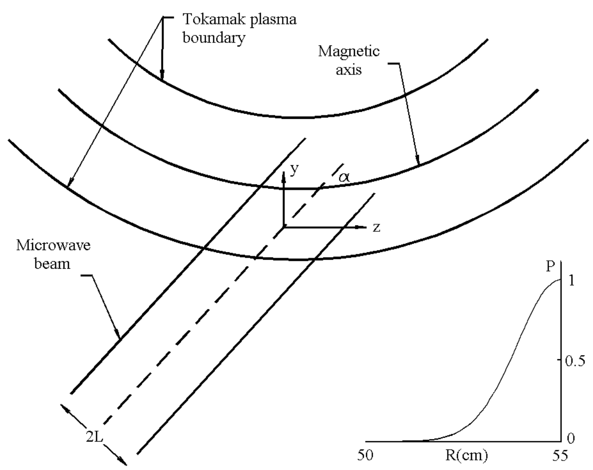

FIG. 1. Plan view of RF injection geometry

So, it is assumed that the microwave beam is injected into a tokamak in the midplane and at an angle α to the magnetic axis. The spatial dependence of the wave field can be written as,

$$E(\vec{r}) = E_0(R)\exp\left\{-y^2/L^2 - z^2\sin^2\alpha/L^2\right\} \tag{6}$$

where the y-axis is out of the plane of the paper, 2L is the width of the injected beam and $E_0(R)$ is the electric field on the beam axis.

If the widths of the wave packet in poloidal and toroidal directions, $\Delta\chi$ and Δz respectively, are not too large, the diffusion coefficient will be a broadened resonance function, picked around the "central" resonance,

$$\gamma - \omega_c(\bar{\chi})/\omega - \bar{N}_{||}x_{||}(\bar{\chi})\mu^{1/2} = 0 \tag{7}$$

where $\bar{\chi}$ denotes the poloidal location of the place where the central ray intersects the magnetic surface, $\bar{N}_{||} = N_{||}(\bar{\chi})$, $x_{||} = \vec{x}\cdot\vec{B}/B$, $\mu = mc^2/2T$, $\gamma = (1+x^2/\mu)^{1/2}$. The width of the resonance is the sum of the geometrical widths of $N_{||}$ and ω_c/ω (and $x_{||}$). This width is the important physical parameter which should be accurately evaluated. In the present paper we shall use the bounce averaged treatment of the quasilinear diffusion operator which has been applied in O'Brien et al. [2]. This procedure incorporates many of the geometrical and physical features encountered in experiments. In particular, it takes account of the finite size of the microwave input beam, the variation of both, the cyclotron frequency and the parallel velocity of the particle as it moves along the field lines, the attenuation of the wave as it propagates and the heating out of resonance of electrons before they can completely cross the microwave beam. This last effect is due to the relativistic mass increase as the electrons gain energy and is particularly important for electrons with a small parallel velocity component.

The expression for the bounce averaged EC quasilinear diffusion coefficient derived in [2] has the following form,

$$D_0 = D_{\perp\perp} = \frac{\pi}{16\tau}\left(\frac{e}{m(n-1)!}\right)^2 \left\{|E_0(R)|^2 \left(\frac{k_\perp v_\perp}{2\omega_c}\right)^{2(n-1)} \frac{e^{-2y^2/L^2}}{W v_\parallel^3}\right\}_{\chi_{res}} \frac{K}{\oint d\chi/v_\parallel} \tag{8}$$

where χ_{res} is the poloidal angle at which the resonance condition

$$\left\{\omega - n\omega_c - k_\parallel v_\parallel\right\}_{\chi_{res}} = 0 \tag{9}$$

is satisfied. Here $W^2 = \sin^4\alpha/L^4 + (n\Omega_t'/2v_\parallel)^2$ while the quantity K is the integral,

$$K = \oint \exp\left\{-\frac{\sin^2\alpha}{2L^2W^2}\left(\frac{\omega - n\omega_c}{v_\parallel} - k_\parallel\right)^2\right\} d\chi \tag{10}$$

The limits of integration are $(-\pi, \pi)$ for passing and $(-\chi_t, \chi_t)$ for trapped electrons.

For low parallel velocities, the electrons remain in the beam long enough to be heated out of resonance, because of the relativistic dependence of the electron mass on energy. This effect serves to limit the diffusion coefficient D_0 to finite values at low parallel velocity. In [2] the following estimate of its value is given,

$$D_H = \frac{2}{\tau}\left(\frac{c^2 v_\parallel}{L\omega v_\perp}\right)^2 \tag{11}$$

III Current generation by electron cyclotron waves

The mechanism of current generation by EC resonant heating relies on an asymmetric alteration of the electron collision rate, generated by waves injected obliquely with respect to the toroidal magnetic field. The means of accomplishing this asymmetric resistivity is selective heating of particles moving in one direction. Being hotter, they naturally collide less. For example, heating of electrons moving to the right would result in a net electric current with ions moving to the left. The interest in driving currents in this manner arises from the possibility of operating tokamak fusion reactors in the steady state by replacing the inherently pulsed Ohmic transformer current. The crucial quantity by which the practicality of this scheme may be assessed is J/P_d, the amount of current generated per power dissipated. The RF driven current density is defined by the following relation,

$$J = -e\int v_\parallel f d^3v \tag{12}$$

while the power dissipated in plasma (or the rate of energy gain by electrons due to the RF waves) is defined as,

$$P_d = \frac{m}{2}\int v^2 \left(\frac{\partial f}{\partial t}\right)_w d^3v \tag{13}$$

High-frequency plasma heating methods including EC plasma heating can be used to create an asymmetric plasma resistivity and drive a net toroidal current. As we shall see later, EC plasma heating is essentially perpendicular to the toroidal direction. It should be noted that the presence of trapped electrons imposes a stringent limit on the efficiency of the current drive mechanism. This happens because electrons are diffused by the EC waves towards large perpendicular velocities (see Fig.5 for example). Thus their distribution tends to be efficiently symmetrized by collisions with trapped particles so that their contribution to the toroidal current is appreciably reduced. Therefore, the current drive efficiency by EC waves absorbed away from the plasma center will be a sensitive function of the detailed shape of the electron DF near the trapped/passing boundary in velocity space. High current drive efficiencies can be attained only if high velocity, poorly collisional electrons are heated ($x_{||} > 4$), a regime in which quasilinear effects are strong and large distortions in the electron DF are easily generated, which can affect the wave damping.

IV Results

The expression describing the flux induced by the quasilinear wave-particle interaction (2) with the bounce-averaged EC quasilinear diffusion coefficient is introduced in the bounce-averaged FP equation (Eq. (20) in [1]). In order to see clearly the effect of the microwave beam, the Ohmic electric field is set to zero ($S_e = 0$).

The quasilinear FP numerical code we have developed has been used to simulate the second harmonic extraordinary (X) mode plasma heating experiment in a medium size tokamak like COMPASS. The geometric and plasma parameters are taken to be: major radius of the tokamak $R_a = 55cm$, plasma radius $\rho_p = 11cm$, the magnetic field on the axis $B_a = 1.0132T$, plasma density $n = 10^{19}m^{-3}$, electron temperature $T = 1.5keV$ and an effective ion charge $Z = 2$. Second harmonic X-mode electron cyclotron waves at frequency $60GHz$, a beam width $L = 5cm$ and at an incident angle of $70°$ to the field lines, are injected into the plasma. The second harmonic electron cyclotron resonance is placed at $R = 52cm$ so that the cutoff due to the relativistic mass shift occurred at the plasma centre, $R = 55cm$. In what follows, we shall represent the variation of the relevant quantities on two magnetic surfaces: an outer surface with an inverse aspect ratio (a) $\varepsilon = 0.08$ corresponding to a radius $\rho = 4.4cm$ and (b) an inner surface with (b) $\varepsilon = 0.015$ corresponding to $\rho = 0.825cm$. The wave absorption was calculated using the ray tracing code and the resulting wave power along the RF beam axis (normalized to the injected power) is shown as a function of the major radius in the inset of Fig.1. In the present paper we have assumed that the represented variation $P(R)$ can be approximated by $P(R) \sim \exp\{-((R - R_a)/1.56cm)^2\}$. Before going on, we recall that in our FP studies we normalize velocities to v_t, times to t_c, the electron density to n, the electron distribution to n/v_t , the quasilinear diffusion coefficient to $v_t\nu$, the electric field to $mv_t\nu/e$, the current density to nev_t, power density to $nmv_t\nu/2$, etc.

IV.1 The bounce-averaged quasilinear diffusion coefficients

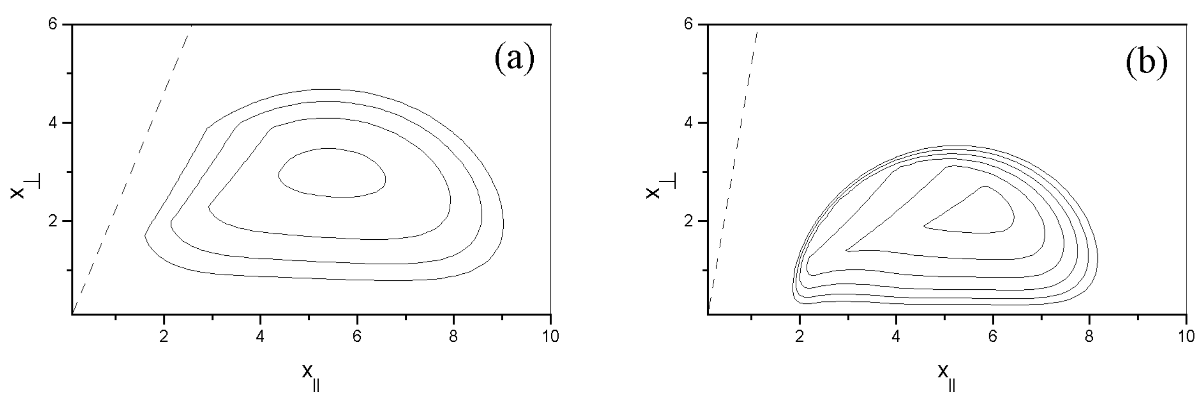

FIG. 2. Quasilinear Diffusion Coefficients for (a) $\varepsilon = 0.08$ *and* (b) $\varepsilon = 0.015$.

The bounce averaged quasilinear diffusion coefficient is determined using the expressions (8) and

(11). More precisely, the smaller of these two expressions is used because the value of $\Delta v_\perp$ corresponding to D_0, for $D_0 > D_H$ would be more than sufficient to heat the electrons out of resonance. On Fig.2 we have represented the contours of bounce averaged quasilinear diffusion coefficient at the afore mentioned magnetic surfaces for an injected RF power of 2MW. The dashed lines indicate the trapped/passing boundary. The outer contours in both cases, correspond to the same diffusion coefficient value $D = 2.31 \cdot 10^{18} m^2/s^3$. Adjacent contours represent factor-of-two change in the diffusion coefficient. Generally, for the considered wave launching ($\alpha = 70°$), very low values of the diffusion coefficient are obtained for the counter-passing particles. Since the values of the diffusion coefficient in the counter-passing region ($x < 0$) are very small ($D_0 < 10^{17} m^2/s^3$), on Fig.2 only the contours in the co-passing velocity region are represented. The resulting diffusion coefficient is a complex function of the velocity space variables. As one can see, the resonance is spread throughout the co-passing velocity space. The maximum diffusion coefficient occurs for large $x_{||}$-values ($x_{||} \approx 5-6$). The maximum value of D is nearly four times larger for the smaller flux surface (case (b)). The decrease of the diffusion coefficient towards the trapped/passing boundary is due to the influence of D_H as the local value of $v_{||}$ tends to zero.

IV.2 The temporal evolution of the power

We have examined the temporal evolution of the power. We distinguish two powers, which, respectively, describe the rate of energy gain by the electrons due to the RF waves P_d (13) and the rate due to collisions with the background distribution

$$P_c = \frac{m}{2} \int v^2 \left(\frac{\partial f}{\partial \tau} \right)_c d^3 v \tag{14}$$

In Fig.3 we show the temporal evolution of these powers for magnetic flux radii (a) $\rho = 4.4cm$ and (b) $\rho = 0.825cm$, corresponding to inverse aspect ratios $\varepsilon = 0.08$ and $\varepsilon = 0.015$, respectively. We recall that $\tau = t\nu$ is the normalized time and ν is the thermal collision frequency. One can see that P_d is initially large but quickly attains its time-asymptotic value. On the other hand, P_c approaches its asymptotic value much more slowly. Note that for the larger flux surface (a) the steady state is attained relatively quickly ($\tau \geq 70$) while for the smaller one (b) it requires much more time ($\tau > 300$). In the steady state the power gained from the RF is to within a few per cents balanced by the power lost to the background distribution that is, we have $P_d \cong -P_c$. The small discrepancy from the exact balance of these powers is due to the fact that the line particle density is not rigorously confined.

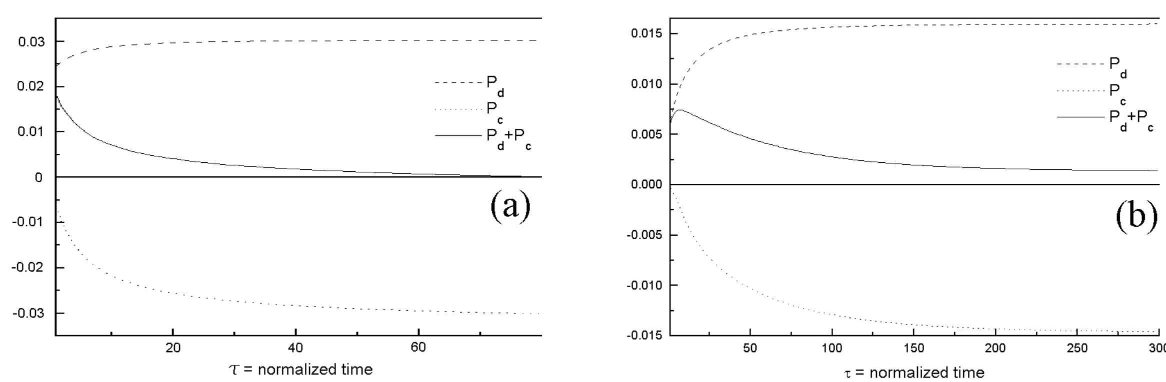

FIG. 3. Temporal Evolution of the DF for (a) $\varepsilon = 0.08$ and (b) $\varepsilon = 0.015$

IV.3 Contours of electron distribution function

The obtained steady state solutions of the bounce-averaged quasilinear FP equation have interesting features. In Fig.4 we show the contours of the steady state electron distribution functions obtained for an injected RF power of $1.5 MW$ and the considered two magnetic surfaces. Dashed lines show the boundaries between the trapped and passing electrons. Fig.4 demonstrates clearly the symmetry of the distribution function in the trapped region of velocity space. It should be noted that the contours are plotted at intervals such that a Maxwellian distribution gives equally spaced semicircles centred at the origin. As one can see, near the origin the contours of constant electron distribution function in velocity space are equally spaced concentric semicircles of a Maxwellian. This is due to the large collision frequency and small diffusion coefficient in this velocity region. For higher perpendicular components of the velocity, the trapping of electrons in the magnetic mirror formed as a result of the poloidal variation of the magnetic field, becomes efficient. In the considered large injected RF power case ($1.5 MW$), there are strong distortions in the passing region due to the ECR heating . The increase of the perpendicular component of the electron velocity (in particular, in the outer magnetic surface) is obvious. On the other side, the magnetic trapping of low-parallel velocity electrons is very efficient in the inner magnetic surface.

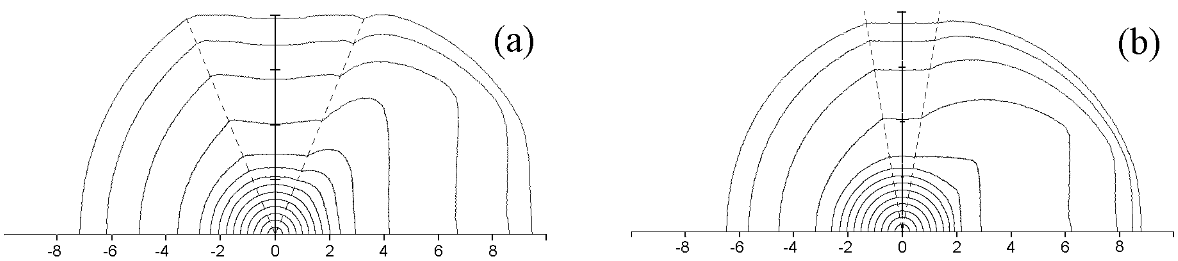

FIG. 4. Steady State DF for (a) $\varepsilon = 0.08$ *and* (b) $\varepsilon = 0.015$

IV.4 RF driven current and absorbed power

In Fig.5 we show the variation of J, P_d, and the current drive efficiency J/P_d with the injected RF power, in the two considered magnetic surfaces: (a) $\rho = 4.4 cm$ and (b) $\rho = 0.825 cm$. The unity of current drive corresponds to values (a) $3.37 \cdot 10^7 A/m^2$ and (b) $3.67 \cdot 10^7 A/m^2$, the unity of power density corresponds to values (a) $5.63 \cdot 10^7 W/m^3$ and (b) $5.18 \cdot 10^7 W/m^3$ and the unity of current drive efficiency corresponds to values (a) $0.60 Am/W$ and (b) $0.71 Am/W$. As one can see, the current density is strongly dependent on the injected RF power. This is a result of the interplay of the effects related to the presence of trapped particles and quasilinear effects, which generate large perpendicular deformations of the DF. At low power levels an important increase of the current density with the injected RF power is observed. At medium power levels this increase is further raised. Finally, at high power levels ($P_{inj} \sim 2 MW$) a saturation of J begins to occur. This is due to the effect of heating out of resonance. Namely, we use the smaller of D_0 (which increases with P_{inj}) and D_H (which is independent on P_{inj}) so that the influence of D_H increases as the injected power is raised. In the variation of the absorbed power density, two characteristic regions can also be distinguished: the low- and high-power one. The increase of P_d with the injected RF power is somewhat smaller. Note for example, that at an injected RF power of 1 MW the absorbed power density is $1.0 MW/m^3$ for case (a). In Fig.5 we have shown also the variation of the current drive efficiency J/P_d with the injected power. It appears that the current drive efficiency which, for a homogeneous magnetic field, is weakly dependent on the injected RF power, becomes strongly power dependent in toroidal geometry. The value of the efficiency in the outer flux surface is smaller than the inner one. This is partly due to the influence of trapping and partly because numerous low energy electrons can come to resonance as the inverse aspect ratio is increased. The current drive efficiency is also defined as,

$$\eta = \frac{J}{2\pi R_a P_d} \tag{15}$$

The current drive efficiency η varies from $0.08 - 0.29 A/W$ in the outer magnetic surface and from

$0.52 - 1.4A/W$ in the inner flux surface.

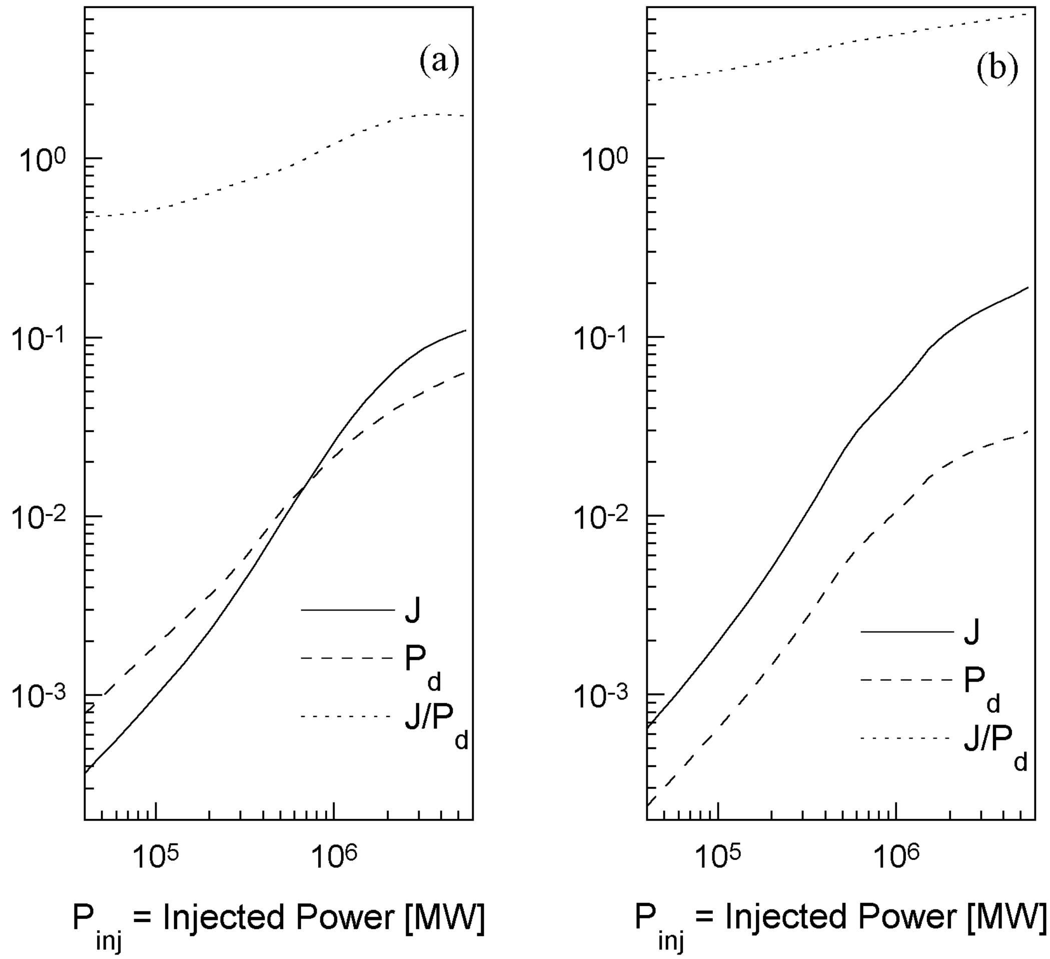

FIG. 5. Current Density, Absorbed Power Density and Current Drive Efficiency for (a) $\varepsilon = 0.08$ *and* (b) $\varepsilon = 0.015$

V Conclusions

The main aim of this work was to point out and simulate some important aspects of the physics of plasma heating and current drive by EC waves in fusion plasmas, analyzing an illustrative tokamak plasma heating experiment. The numerical code we have developed to solve the bounce-averaged quasilinear Fokker-Planck equation is an interpretative and predictive code. It incorporates many of the geometrical and physical features encountered in experiments. In particular, it takes account of the finite size of the microwave input beam, the variation of both, the cyclotron frequency and the parallel velocity of the particle as it moves along the field lines, the attenuation of the wave as it propagates and the heating out of resonance of electrons before they can completely cross the microwave beam. We found that the EC quasilinear diffusion coefficient is a complex function of the velocity space variables. For the considered wave launching, the resonance is spread throughout the co-passing velocity space while the maximum diffusion coefficient occurs for large, poorly collisional electrons $x_{||}$-values ($x_{||} \approx 5 - 6$). In this regime, quasilinear effects are strong and, due to the ECR plasma heating, large distortions in the electron DF are generated. The RF driven current density is strongly dependent on the injected power. This is a result of the interplay of the effects related to the presence of trapped particles and quasilinear effects, which generate large perpendicular deformations of the DF. At high power levels ($P_{inj} \sim 2MW$) a saturation of the current density, which is due to the effect of heating out of resonance begins to occur. The increase of the dissipated RF power with the injected power is somewhat smaller than the current density one. A deterioration of the current drive efficiency caused by the interplay of electron trapping and quasilinear effects, is observed. These results agree with the recent RF current drive studies in tokamak plasma.

The main conclusion of this analysis is that for the wave power levels relevant to present-day and future experiments, the process of current drive by EC waves is dominated by quasilinear and toroidal effects.

References

[1] S.Pešić and I.Sajić, Publications of the Faculty of Electrical Engineering, Engineering Physics (1996) 35.

[2] M.R.O'Brien, M.Cox and D.F.H.Start, Nucl. Fusion **26** (1986) 1625.

(Received June 1998.)